## Chapter 111. Texas Essential Knowledge and Skills for Mathematics

## Subchapter A. Elementary

Statutory Authority: The provisions of this Subchapter A issued under the Texas Education Code, §7.102(c)(4), 28.002, and 28.0021(a)(1), unless otherwise noted.

# **§111.1. Implementation of Texas Essential Knowledge and Skills for Mathematics, Elementary, Adopted 2012.**

- (a) The provisions of §§111.2-111.7 of this subchapter shall be implemented by school districts.
- (b) No later than August 31, 2013, the commissioner of education shall determine whether instructional materials funding has been made available to Texas public schools for materials that cover the essential knowledge and skills for mathematics as adopted in §§111.2-111.7 of this subchapter.
- (c) If the commissioner makes the determination that instructional materials funding has been made available under subsection (b) of this section, §§111.2-111.7 of this subchapter shall be implemented beginning with the 2014-2015 school year and apply to the 2014-2015 and subsequent school years.
- (d) If the commissioner does not make the determination that instructional materials funding has been made available under subsection (b) of this section, the commissioner shall determine no later than August 31 of each subsequent school year whether instructional materials funding has been made available. If the commissioner determines that instructional materials funding has been made available, the commissioner shall notify the State Board of Education and school districts that §§111.2-111.7 of this subchapter shall be implemented for the following school year.
- (e) Sections 111.11-111.17 of this subchapter shall be superseded by the implementation of §§111.1-111.7 under this section.

Source: The provisions of this §111.1 adopted to be effective September 10, 2012, 37 TexReg 7109.

## §111.2. Kindergarten, Adopted 2012.

- (a) Introduction.
  - (1) The desire to achieve educational excellence is the driving force behind the Texas essential knowledge and skills for mathematics, guided by the college and career readiness standards. By embedding statistics, probability, and finance, while focusing on computational thinking, mathematical fluency, and solid understanding, Texas will lead the way in mathematics education and prepare all Texas students for the challenges they will face in the 21st century.
  - (2)The process standards describe ways in which students are expected to engage in the content. The placement of the process standards at the beginning of the knowledge and skills listed for each grade and course is intentional. The process standards weave the other knowledge and skills together so that students may be successful problem solvers and use mathematics efficiently and effectively in daily life. The process standards are integrated at every grade level and course. When possible, students will apply mathematics to problems arising in everyday life, society, and the workplace. Students will use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution. Students will select appropriate tools such as real objects, manipulatives, algorithms, paper and pencil, and technology and techniques such as mental math, estimation, number sense, and generalization and abstraction to solve problems. Students will effectively communicate mathematical ideas, reasoning, and their implications using multiple representations such as symbols, diagrams, graphs, computer programs, and language. Students will use mathematical relationships to generate solutions and make connections and predictions. Students will analyze mathematical relationships to connect and communicate mathematical ideas. Students will display, explain, or justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

- (3) For students to become fluent in mathematics, students must develop a robust sense of number. The National Research Council's report, "Adding It Up," defines procedural fluency as "skill in carrying out procedures flexibly, accurately, efficiently, and appropriately." As students develop procedural fluency, they must also realize that true problem solving may take time, effort, and perseverance. Students in Kindergarten are expected to perform their work without the use of calculators.
- (4) The primary focal areas in Kindergarten are understanding counting and cardinality, understanding addition as joining and subtraction as separating, and comparing objects by measurable attributes.
  - (A) Students develop number and operations through several fundamental concepts. Students know number names and the counting sequence. Counting and cardinality lay a solid foundation for number. Students apply the principles of counting to make the connection between numbers and quantities.
  - (B) Students use meanings of numbers to create strategies for solving problems and responding to practical situations involving addition and subtraction.
  - (C) Students identify characteristics of objects that can be measured and directly compare objects according to these measurable attributes.
- (5) Statements that contain the word "including" reference content that must be mastered, while those containing the phrase "such as" are intended as possible illustrative examples.
- (b) Knowledge and skills.
  - (1) Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:
    - (A) apply mathematics to problems arising in everyday life, society, and the workplace;
    - (B) use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;
    - select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;
    - (D) communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;
    - (E) create and use representations to organize, record, and communicate mathematical ideas;
    - (F) analyze mathematical relationships to connect and communicate mathematical ideas; and
    - (G) display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.
  - (2) Number and operations. The student applies mathematical process standards to understand how to represent and compare whole numbers, the relative position and magnitude of whole numbers, and relationships within the numeration system. The student is expected to:
    - (A) count forward and backward to at least 20 with and without objects;
    - (B) read, write, and represent whole numbers from 0 to at least 20 with and without objects or pictures;
    - (C) count a set of objects up to at least 20 and demonstrate that the last number said tells the number of objects in the set regardless of their arrangement or order;
    - (D) recognize instantly the quantity of a small group of objects in organized and random arrangements;

- (E) generate a set using concrete and pictorial models that represents a number that is more than, less than, and equal to a given number up to 20;
- (F) generate a number that is one more than or one less than another number up to at least 20;
- (G) compare sets of objects up to at least 20 in each set using comparative language;
- (H) use comparative language to describe two numbers up to 20 presented as written numerals; and
- (I) compose and decompose numbers up to 10 with objects and pictures.
- (3) Number and operations. The student applies mathematical process standards to develop an understanding of addition and subtraction situations in order to solve problems. The student is expected to:
  - (A) model the action of joining to represent addition and the action of separating to represent subtraction;
  - (B) solve word problems using objects and drawings to find sums up to 10 and differences within 10; and
  - (C) explain the strategies used to solve problems involving adding and subtracting within 10 using spoken words, concrete and pictorial models, and number sentences.
- (4) Number and operations. The student applies mathematical process standards to identify coins in order to recognize the need for monetary transactions. The student is expected to identify U.S. coins by name, including pennies, nickels, dimes, and quarters.
- (5) Algebraic reasoning. The student applies mathematical process standards to identify the pattern in the number word list. The student is expected to recite numbers up to at least 100 by ones and tens beginning with any given number.
- (6) Geometry and measurement. The student applies mathematical process standards to analyze attributes of two-dimensional shapes and three-dimensional solids to develop generalizations about their properties. The student is expected to:
  - (A) identify two-dimensional shapes, including circles, triangles, rectangles, and squares as special rectangles;
  - (B) identify three-dimensional solids, including cylinders, cones, spheres, and cubes, in the real world;
  - (C) identify two-dimensional components of three-dimensional objects;
  - (D) identify attributes of two-dimensional shapes using informal and formal geometric language interchangeably;
  - (E) classify and sort a variety of regular and irregular two- and three-dimensional figures regardless of orientation or size; and
  - (F) create two-dimensional shapes using a variety of materials and drawings.
- (7) Geometry and measurement. The student applies mathematical process standards to directly compare measurable attributes. The student is expected to:
  - (A) give an example of a measurable attribute of a given object, including length, capacity, and weight; and
  - (B) compare two objects with a common measurable attribute to see which object has more of/less of the attribute and describe the difference.
- (8) Data analysis. The student applies mathematical process standards to collect and organize data to make it useful for interpreting information. The student is expected to:
  - (A) collect, sort, and organize data into two or three categories;

- (B) use data to create real-object and picture graphs; and
- (C) draw conclusions from real-object and picture graphs.
- (9) Personal financial literacy. The student applies mathematical process standards to manage one's financial resources effectively for lifetime financial security. The student is expected to:
  - (A) identify ways to earn income;
  - (B) differentiate between money received as income and money received as gifts;
  - (C) list simple skills required for jobs; and
  - (D) distinguish between wants and needs and identify income as a source to meet one's wants and needs.

Source: The provisions of this \$111.2 adopted to be effective September 10, 2012, 37 TexReg 7109.

#### §111.3. Grade 1, Adopted 2012.

- (a) Introduction.
  - (1) The desire to achieve educational excellence is the driving force behind the Texas essential knowledge and skills for mathematics, guided by the college and career readiness standards. By embedding statistics, probability, and finance, while focusing on computational thinking, mathematical fluency, and solid understanding, Texas will lead the way in mathematics education and prepare all Texas students for the challenges they will face in the 21st century.
  - (2)The process standards describe ways in which students are expected to engage in the content. The placement of the process standards at the beginning of the knowledge and skills listed for each grade and course is intentional. The process standards weave the other knowledge and skills together so that students may be successful problem solvers and use mathematics efficiently and effectively in daily life. The process standards are integrated at every grade level and course. When possible, students will apply mathematics to problems arising in everyday life, society, and the workplace. Students will use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution. Students will select appropriate tools such as real objects, manipulatives, algorithms, paper and pencil, and technology and techniques such as mental math, estimation, number sense, and generalization and abstraction to solve problems. Students will effectively communicate mathematical ideas, reasoning, and their implications using multiple representations such as symbols, diagrams, graphs, computer programs, and language. Students will use mathematical relationships to generate solutions and make connections and predictions. Students will analyze mathematical relationships to connect and communicate mathematical ideas. Students will display, explain, or justify mathematical ideas and arguments using precise mathematical language in written or oral communication.
  - (3) For students to become fluent in mathematics, students must develop a robust sense of number. The National Research Council's report, "Adding It Up," defines procedural fluency as "skill in carrying out procedures flexibly, accurately, efficiently, and appropriately." As students develop procedural fluency, they must also realize that true problem solving may take time, effort, and perseverance. Students in Grade 1 are expected to perform their work without the use of calculators.
  - (4) The primary focal areas in Grade 1 are understanding and applying place value, solving problems involving addition and subtraction, and composing and decomposing two-dimensional shapes and three-dimensional solids.
    - (A) Students use relationships within the numeration system to understand the sequential order of the counting numbers and their relative magnitude.
    - (B) Students extend their use of addition and subtraction beyond the actions of joining and separating to include comparing and combining. Students use properties of operations and the relationship between addition and subtraction to solve problems. By comparing a

variety of solution strategies, students use efficient, accurate, and generalizable methods to perform operations.

- (C) Students use basic shapes and spatial reasoning to model objects in their environment and construct more complex shapes. Students are able to identify, name, and describe basic two-dimensional shapes and three-dimensional solids.
- (5) Statements that contain the word "including" reference content that must be mastered, while those containing the phrase "such as" are intended as possible illustrative examples.
- (b) Knowledge and skills.
  - (1) Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:
    - (A) apply mathematics to problems arising in everyday life, society, and the workplace;
    - (B) use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;
    - select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;
    - (D) communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;
    - (E) create and use representations to organize, record, and communicate mathematical ideas;
    - (F) analyze mathematical relationships to connect and communicate mathematical ideas; and
    - (G) display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.
  - (2) Number and operations. The student applies mathematical process standards to represent and compare whole numbers, the relative position and magnitude of whole numbers, and relationships within the numeration system related to place value. The student is expected to:
    - (A) recognize instantly the quantity of structured arrangements;
    - (B) use concrete and pictorial models to compose and decompose numbers up to 120 in more than one way as so many hundreds, so many tens, and so many ones;
    - (C) use objects, pictures, and expanded and standard forms to represent numbers up to 120;
    - (D) generate a number that is greater than or less than a given whole number up to 120;
    - (E) use place value to compare whole numbers up to 120 using comparative language;
    - (F) order whole numbers up to 120 using place value and open number lines; and
    - (G) represent the comparison of two numbers to 100 using the symbols >, <, or =.
  - (3) Number and operations. The student applies mathematical process standards to develop and use strategies for whole number addition and subtraction computations in order to solve problems. The student is expected to:
    - (A) use concrete and pictorial models to determine the sum of a multiple of 10 and a one-digit number in problems up to 99;
    - (B) use objects and pictorial models to solve word problems involving joining, separating, and comparing sets within 20 and unknowns as any one of the terms in the problem such as 2 + 4 = []; 3 + [] = 7; and 5 = [] 3;
    - (C) compose 10 with two or more addends with and without concrete objects;

- (D) apply basic fact strategies to add and subtract within 20, including making 10 and decomposing a number leading to a 10;
- (E) explain strategies used to solve addition and subtraction problems up to 20 using spoken words, objects, pictorial models, and number sentences; and
- (F) generate and solve problem situations when given a number sentence involving addition or subtraction of numbers within 20.
- (4) Number and operations. The student applies mathematical process standards to identify coins, their values, and the relationships among them in order to recognize the need for monetary transactions. The student is expected to:
  - (A) identify U.S. coins, including pennies, nickels, dimes, and quarters, by value and describe the relationships among them;
  - (B) write a number with the cent symbol to describe the value of a coin; and
  - (C) use relationships to count by twos, fives, and tens to determine the value of a collection of pennies, nickels, and/or dimes.
- (5) Algebraic reasoning. The student applies mathematical process standards to identify and apply number patterns within properties of numbers and operations in order to describe relationships. The student is expected to:
  - (A) recite numbers forward and backward from any given number between 1 and 120;
  - (B) skip count by twos, fives, and tens to determine the total number of objects up to 120 in a set;
  - (C) use relationships to determine the number that is 10 more and 10 less than a given number up to 120;
  - (D) represent word problems involving addition and subtraction of whole numbers up to 20 using concrete and pictorial models and number sentences;
  - (E) understand that the equal sign represents a relationship where expressions on each side of the equal sign represent the same value(s);
  - (F) determine the unknown whole number in an addition or subtraction equation when the unknown may be any one of the three or four terms in the equation; and
  - (G) apply properties of operations to add and subtract two or three numbers.
- (6) Geometry and measurement. The student applies mathematical process standards to analyze attributes of two-dimensional shapes and three-dimensional solids to develop generalizations about their properties. The student is expected to:
  - (A) classify and sort regular and irregular two-dimensional shapes based on attributes using informal geometric language;
  - (B) distinguish between attributes that define a two-dimensional or three-dimensional figure and attributes that do not define the shape;
  - (C) create two-dimensional figures, including circles, triangles, rectangles, and squares, as special rectangles, rhombuses, and hexagons;
  - (D) identify two-dimensional shapes, including circles, triangles, rectangles, and squares, as special rectangles, rhombuses, and hexagons and describe their attributes using formal geometric language;
  - (E) identify three-dimensional solids, including spheres, cones, cylinders, rectangular prisms (including cubes), and triangular prisms, and describe their attributes using formal geometric language;

- (F) compose two-dimensional shapes by joining two, three, or four figures to produce a target shape in more than one way if possible;
- (G) partition two-dimensional figures into two and four fair shares or equal parts and describe the parts using words; and
- (H) identify examples and non-examples of halves and fourths.
- (7) Geometry and measurement. The student applies mathematical process standards to select and use units to describe length and time. The student is expected to:
  - (A) use measuring tools to measure the length of objects to reinforce the continuous nature of linear measurement;
  - (B) illustrate that the length of an object is the number of same-size units of length that, when laid end-to-end with no gaps or overlaps, reach from one end of the object to the other;
  - (C) measure the same object/distance with units of two different lengths and describe how and why the measurements differ;
  - (D) describe a length to the nearest whole unit using a number and a unit; and
  - (E) tell time to the hour and half hour using analog and digital clocks.
- (8) Data analysis. The student applies mathematical process standards to organize data to make it useful for interpreting information and solving problems. The student is expected to:
  - (A) collect, sort, and organize data in up to three categories using models/representations such as tally marks or T-charts;
  - (B) use data to create picture and bar-type graphs; and
  - (C) draw conclusions and generate and answer questions using information from picture and bar-type graphs.
- (9) Personal financial literacy. The student applies mathematical process standards to manage one's financial resources effectively for lifetime financial security. The student is expected to:
  - (A) define money earned as income;
  - (B) identify income as a means of obtaining goods and services, oftentimes making choices between wants and needs;
  - (C) distinguish between spending and saving; and
  - (D) consider charitable giving.

Source: The provisions of this §111.3 adopted to be effective September 10, 2012, 37 TexReg 7109.

## §111.4. Grade 2, Adopted 2012.

- (a) Introduction.
  - (1) The desire to achieve educational excellence is the driving force behind the Texas essential knowledge and skills for mathematics, guided by the college and career readiness standards. By embedding statistics, probability, and finance, while focusing on computational thinking, mathematical fluency, and solid understanding, Texas will lead the way in mathematics education and prepare all Texas students for the challenges they will face in the 21st century.
  - (2) The process standards describe ways in which students are expected to engage in the content. The placement of the process standards at the beginning of the knowledge and skills listed for each grade and course is intentional. The process standards weave the other knowledge and skills together so that students may be successful problem solvers and use mathematics efficiently and effectively in daily life. The process standards are integrated at every grade level and course. When possible, students will apply mathematics to problems arising in everyday life, society, and the workplace. Students will use a problem-solving model that incorporates analyzing given

information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution. Students will select appropriate tools such as real objects, manipulatives, algorithms, paper and pencil, and technology and techniques such as mental math, estimation, number sense, and generalization and abstraction to solve problems. Students will effectively communicate mathematical ideas, reasoning, and their implications using multiple representations such as symbols, diagrams, graphs, computer programs, and language. Students will use mathematical relationships to generate solutions and make connections and predictions. Students will analyze mathematical relationships to connect and communicate mathematical ideas. Students will display, explain, or justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

- (3) For students to become fluent in mathematics, students must develop a robust sense of number. The National Research Council's report, "Adding It Up," defines procedural fluency as "skill in carrying out procedures flexibly, accurately, efficiently, and appropriately." As students develop procedural fluency, they must also realize that true problem solving may take time, effort, and perseverance. Students in Grade 2 are expected to perform their work without the use of calculators.
- (4) The primary focal areas in Grade 2 are making comparisons within the base-10 place value system, solving problems with addition and subtraction within 1,000, and building foundations for multiplication.
  - (A) Students develop an understanding of the base-10 place value system and place value concepts. The students' understanding of base-10 place value includes ideas of counting in units and multiples of thousands, hundreds, tens, and ones and a grasp of number relationships, which students demonstrate in a variety of ways.
  - (B) Students identify situations in which addition and subtraction are useful to solve problems. Students develop a variety of strategies to use efficient, accurate, and generalizable methods to add and subtract multi-digit whole numbers.
  - (C) Students use the relationship between skip counting and equal groups of objects to represent the addition or subtraction of equivalent sets, which builds a strong foundation for multiplication and division.
- (5) Statements that contain the word "including" reference content that must be mastered, while those containing the phrase "such as" are intended as possible illustrative examples.

#### (b) Knowledge and skills.

- (1) Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:
  - (A) apply mathematics to problems arising in everyday life, society, and the workplace;
  - (B) use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;
  - select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;
  - (D) communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;
  - (E) create and use representations to organize, record, and communicate mathematical ideas;
  - (F) analyze mathematical relationships to connect and communicate mathematical ideas; and
  - (G) display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

- (2) Number and operations. The student applies mathematical process standards to understand how to represent and compare whole numbers, the relative position and magnitude of whole numbers, and relationships within the numeration system related to place value. The student is expected to:
  - (A) use concrete and pictorial models to compose and decompose numbers up to 1,200 in more than one way as a sum of so many thousands, hundreds, tens, and ones;
  - (B) use standard, word, and expanded forms to represent numbers up to 1,200;
  - (C) generate a number that is greater than or less than a given whole number up to 1,200;
  - (D) use place value to compare and order whole numbers up to 1,200 using comparative language, numbers, and symbols (>, <, or =);
  - (E) locate the position of a given whole number on an open number line; and
  - (F) name the whole number that corresponds to a specific point on a number line.
- (3) Number and operations. The student applies mathematical process standards to recognize and represent fractional units and communicates how they are used to name parts of a whole. The student is expected to:
  - (A) partition objects into equal parts and name the parts, including halves, fourths, and eighths, using words;
  - (B) explain that the more fractional parts used to make a whole, the smaller the part; and the fewer the fractional parts, the larger the part;
  - (C) use concrete models to count fractional parts beyond one whole using words and recognize how many parts it takes to equal one whole; and
  - (D) identify examples and non-examples of halves, fourths, and eighths.
- (4) Number and operations. The student applies mathematical process standards to develop and use strategies and methods for whole number computations in order to solve addition and subtraction problems with efficiency and accuracy. The student is expected to:
  - (A) recall basic facts to add and subtract within 20 with automaticity;
  - (B) add up to four two-digit numbers and subtract two-digit numbers using mental strategies and algorithms based on knowledge of place value and properties of operations;
  - (C) solve one-step and multi-step word problems involving addition and subtraction within 1,000 using a variety of strategies based on place value, including algorithms; and
  - (D) generate and solve problem situations for a given mathematical number sentence involving addition and subtraction of whole numbers within 1,000.
- (5) Number and operations. The student applies mathematical process standards to determine the value of coins in order to solve monetary transactions. The student is expected to:
  - (A) determine the value of a collection of coins up to one dollar; and
  - (B) use the cent symbol, dollar sign, and the decimal point to name the value of a collection of coins.
- (6) Number and operations. The student applies mathematical process standards to connect repeated addition and subtraction to multiplication and division situations that involve equal groupings and shares. The student is expected to:
  - (A) model, create, and describe contextual multiplication situations in which equivalent sets of concrete objects are joined; and
  - (B) model, create, and describe contextual division situations in which a set of concrete objects is separated into equivalent sets.

- (7) Algebraic reasoning. The student applies mathematical process standards to identify and apply number patterns within properties of numbers and operations in order to describe relationships. The student is expected to:
  - (A) determine whether a number up to 40 is even or odd using pairings of objects to represent the number;
  - (B) use an understanding of place value to determine the number that is 10 or 100 more or less than a given number up to 1,200; and
  - (C) represent and solve addition and subtraction word problems where unknowns may be any one of the terms in the problem.
- (8) Geometry and measurement. The student applies mathematical process standards to analyze attributes of two-dimensional shapes and three-dimensional solids to develop generalizations about their properties. The student is expected to:
  - (A) create two-dimensional shapes based on given attributes, including number of sides and vertices;
  - (B) classify and sort three-dimensional solids, including spheres, cones, cylinders, rectangular prisms (including cubes as special rectangular prisms), and triangular prisms, based on attributes using formal geometric language;
  - (C) classify and sort polygons with 12 or fewer sides according to attributes, including identifying the number of sides and number of vertices;
  - (D) compose two-dimensional shapes and three-dimensional solids with given properties or attributes; and
  - (E) decompose two-dimensional shapes such as cutting out a square from a rectangle, dividing a shape in half, or partitioning a rectangle into identical triangles and identify the resulting geometric parts.
- (9) Geometry and measurement. The student applies mathematical process standards to select and use units to describe length, area, and time. The student is expected to:
  - (A) find the length of objects using concrete models for standard units of length;
  - (B) describe the inverse relationship between the size of the unit and the number of units needed to equal the length of an object;
  - (C) represent whole numbers as distances from any given location on a number line;
  - (D) determine the length of an object to the nearest marked unit using rulers, yardsticks, meter sticks, or measuring tapes;
  - (E) determine a solution to a problem involving length, including estimating lengths;
  - (F) use concrete models of square units to find the area of a rectangle by covering it with no gaps or overlaps, counting to find the total number of square units, and describing the measurement using a number and the unit; and
  - (G) read and write time to the nearest one-minute increment using analog and digital clocks and distinguish between a.m. and p.m.
- (10) Data analysis. The student applies mathematical process standards to organize data to make it useful for interpreting information and solving problems. The student is expected to:
  - (A) explain that the length of a bar in a bar graph or the number of pictures in a pictograph represents the number of data points for a given category;
  - (B) organize a collection of data with up to four categories using pictographs and bar graphs with intervals of one or more;

- (C) write and solve one-step word problems involving addition or subtraction using data represented within pictographs and bar graphs with intervals of one; and
- (D) draw conclusions and make predictions from information in a graph.
- (11) Personal financial literacy. The student applies mathematical process standards to manage one's financial resources effectively for lifetime financial security. The student is expected to:
  - (A) calculate how money saved can accumulate into a larger amount over time;
  - (B) explain that saving is an alternative to spending;
  - (C) distinguish between a deposit and a withdrawal;
  - (D) identify examples of borrowing and distinguish between responsible and irresponsible borrowing;
  - (E) identify examples of lending and use concepts of benefits and costs to evaluate lending decisions; and
  - (F) differentiate between producers and consumers and calculate the cost to produce a simple item.

Source: The provisions of this §111.4 adopted to be effective September 10, 2012, 37 TexReg 7109.

## §111.5. Grade 3, Adopted 2012.

#### (a) Introduction.

- (1) The desire to achieve educational excellence is the driving force behind the Texas essential knowledge and skills for mathematics, guided by the college and career readiness standards. By embedding statistics, probability, and finance, while focusing on computational thinking, mathematical fluency, and solid understanding, Texas will lead the way in mathematics education and prepare all Texas students for the challenges they will face in the 21st century.
- (2)The process standards describe ways in which students are expected to engage in the content. The placement of the process standards at the beginning of the knowledge and skills listed for each grade and course is intentional. The process standards weave the other knowledge and skills together so that students may be successful problem solvers and use mathematics efficiently and effectively in daily life. The process standards are integrated at every grade level and course. When possible, students will apply mathematics to problems arising in everyday life, society, and the workplace. Students will use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution. Students will select appropriate tools such as real objects, manipulatives, algorithms, paper and pencil, and technology and techniques such as mental math, estimation, number sense, and generalization and abstraction to solve problems. Students will effectively communicate mathematical ideas, reasoning, and their implications using multiple representations such as symbols, diagrams, graphs, computer programs, and language. Students will use mathematical relationships to generate solutions and make connections and predictions. Students will analyze mathematical relationships to connect and communicate mathematical ideas. Students will display, explain, or justify mathematical ideas and arguments using precise mathematical language in written or oral communication.
- (3) For students to become fluent in mathematics, students must develop a robust sense of number. The National Research Council's report, "Adding It Up," defines procedural fluency as "skill in carrying out procedures flexibly, accurately, efficiently, and appropriately." As students develop procedural fluency, they must also realize that true problem solving may take time, effort, and perseverance. Students in Grade 3 are expected to perform their work without the use of calculators.
- (4) The primary focal areas in Grade 3 are place value, operations of whole numbers, and understanding fractional units. These focal areas are supported throughout the mathematical strands of number and operations, algebraic reasoning, geometry and measurement, and data

analysis. In Grades 3-5, the number set is limited to positive rational numbers. In number and operations, students will focus on applying place value, comparing and ordering whole numbers, connecting multiplication and division, and understanding and representing fractions as numbers and equivalent fractions. In algebraic reasoning, students will use multiple representations of problem situations, determine missing values in number sentences, and represent real-world relationships using number pairs in a table and verbal descriptions. In geometry and measurement, students will identify and classify two-dimensional figures according to common attributes, decompose composite figures formed by rectangles to determine area, determine the perimeter of polygons, solve problems involving time, and measure liquid volume (capacity) or weight. In data analysis, students will represent and interpret data.

(5) Statements that contain the word "including" reference content that must be mastered, while those containing the phrase "such as" are intended as possible illustrative examples.

#### (b) Knowledge and skills.

- (1) Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:
  - (A) apply mathematics to problems arising in everyday life, society, and the workplace;
  - (B) use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;
  - select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;
  - (D) communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;
  - (E) create and use representations to organize, record, and communicate mathematical ideas;
  - (F) analyze mathematical relationships to connect and communicate mathematical ideas; and
  - (G) display, explain, and justify\_mathematical ideas and arguments using precise mathematical language in written or oral communication.
- (2) Number and operations. The student applies mathematical process standards to represent and compare whole numbers and understand relationships related to place value. The student is expected to:
  - (A) compose and decompose numbers up to 100,000 as a sum of so many ten thousands, so many thousands, so many hundreds, so many tens, and so many ones using objects, pictorial models, and numbers, including expanded notation as appropriate;
  - (B) describe the mathematical relationships found in the base-10 place value system through the hundred thousands place;
  - (C) represent a number on a number line as being between two consecutive multiples of 10; 100; 1,000; or 10,000 and use words to describe relative size of numbers in order to round whole numbers; and
  - (D) compare and order whole numbers up to 100,000 and represent comparisons using the symbols >, <, or =.
- (3) Number and operations. The student applies mathematical process standards to represent and explain fractional units. The student is expected to:
  - (A) represent fractions greater than zero and less than or equal to one with denominators of 2, 3, 4, 6, and 8 using concrete objects and pictorial models, including strip diagrams and number lines;

- (B) determine the corresponding fraction greater than zero and less than or equal to one with denominators of 2, 3, 4, 6, and 8 given a specified point on a number line;
- (C) explain that the unit fraction 1/b represents the quantity formed by one part of a whole that has been partitioned into *b* equal parts where *b* is a non-zero whole number;
- (D) compose and decompose a fraction a/b with a numerator greater than zero and less than or equal to b as a sum of parts 1/b;
- (E) solve problems involving partitioning an object or a set of objects among two or more recipients using pictorial representations of fractions with denominators of 2, 3, 4, 6, and 8;
- (F) represent equivalent fractions with denominators of 2, 3, 4, 6, and 8 using a variety of objects and pictorial models, including number lines;
- (G) explain that two fractions are equivalent if and only if they are both represented by the same point on the number line or represent the same portion of a same size whole for an area model; and
- (H) compare two fractions having the same numerator or denominator in problems by reasoning about their sizes and justifying the conclusion using symbols, words, objects, and pictorial models.
- (4) Number and operations. The student applies mathematical process standards to develop and use strategies and methods for whole number computations in order to solve problems with efficiency and accuracy. The student is expected to:
  - (A) solve with fluency one-step and two-step problems involving addition and subtraction within 1,000 using strategies based on place value, properties of operations, and the relationship between addition and subtraction;
  - (B) round to the nearest 10 or 100 or use compatible numbers to estimate solutions to addition and subtraction problems;
  - (C) determine the value of a collection of coins and bills;
  - (D) determine the total number of objects when equally-sized groups of objects are combined or arranged in arrays up to 10 by 10;
  - (E) represent multiplication facts by using a variety of approaches such as repeated addition, equal-sized groups, arrays, area models, equal jumps on a number line, and skip counting;
  - (F) recall facts to multiply up to 10 by 10 with automaticity and recall the corresponding division facts;
  - (G) use strategies and algorithms, including the standard algorithm, to multiply a two-digit number by a one-digit number. Strategies may include mental math, partial products, and the commutative, associative, and distributive properties;
  - (H) determine the number of objects in each group when a set of objects is partitioned into equal shares or a set of objects is shared equally;
  - (I) determine if a number is even or odd using divisibility rules;
  - (J) determine a quotient using the relationship between multiplication and division; and
  - (K) solve one-step and two-step problems involving multiplication and division within 100 using strategies based on objects; pictorial models, including arrays, area models, and equal groups; properties of operations; or recall of facts.
- (5) Algebraic reasoning. The student applies mathematical process standards to analyze and create patterns and relationships. The student is expected to:

- (A) represent one- and two-step problems involving addition and subtraction of whole numbers to 1,000 using pictorial models, number lines, and equations;
- (B) represent and solve one- and two-step multiplication and division problems within 100 using arrays, strip diagrams, and equations;
- (C) describe a multiplication expression as a comparison such as 3 x 24 represents 3 times as much as 24;
- (D) determine the unknown whole number in a multiplication or division equation relating three whole numbers when the unknown is either a missing factor or product; and
- (E) represent real-world relationships using number pairs in a table and verbal descriptions.
- (6) Geometry and measurement. The student applies mathematical process standards to analyze attributes of two-dimensional geometric figures to develop generalizations about their properties. The student is expected to:
  - (A) classify and sort two- and three-dimensional figures, including cones, cylinders, spheres, triangular and rectangular prisms, and cubes, based on attributes using formal geometric language;
  - (B) use attributes to recognize rhombuses, parallelograms, trapezoids, rectangles, and squares as examples of quadrilaterals and draw examples of quadrilaterals that do not belong to any of these subcategories;
  - (C) determine the area of rectangles with whole number side lengths in problems using multiplication related to the number of rows times the number of unit squares in each row;
  - (D) decompose composite figures formed by rectangles into non-overlapping rectangles to determine the area of the original figure using the additive property of area; and
  - (E) decompose two congruent two-dimensional figures into parts with equal areas and express the area of each part as a unit fraction of the whole and recognize that equal shares of identical wholes need not have the same shape.
- (7) Geometry and measurement. The student applies mathematical process standards to select appropriate units, strategies, and tools to solve problems involving customary and metric measurement. The student is expected to:
  - (A) represent fractions of halves, fourths, and eighths as distances from zero on a number line;
  - (B) determine the perimeter of a polygon or a missing length when given perimeter and remaining side lengths in problems;
  - (C) determine the solutions to problems involving addition and subtraction of time intervals in minutes using pictorial models or tools such as a 15-minute event plus a 30-minute event equals 45 minutes;
  - (D) determine when it is appropriate to use measurements of liquid volume (capacity) or weight; and
  - (E) determine liquid volume (capacity) or weight using appropriate units and tools.
- (8) Data analysis. The student applies mathematical process standards to solve problems by collecting, organizing, displaying, and interpreting data. The student is expected to:
  - (A) summarize a data set with multiple categories using a frequency table, dot plot, pictograph, or bar graph with scaled intervals; and
  - (B) solve one- and two-step problems using categorical data represented with a frequency table, dot plot, pictograph, or bar graph with scaled intervals.

- (9) Personal financial literacy. The student applies mathematical process standards to manage one's financial resources effectively for lifetime financial security. The student is expected to:
  - (A) explain the connection between human capital/labor and income;
  - (B) describe the relationship between the availability or scarcity of resources and how that impacts cost;
  - (C) identify the costs and benefits of planned and unplanned spending decisions;
  - (D) explain that credit is used when wants or needs exceed the ability to pay and that it is the borrower's responsibility to pay it back to the lender, usually with interest;
  - (E) list reasons to save and explain the benefit of a savings plan, including for college; and
  - (F) identify decisions involving income, spending, saving, credit, and charitable giving.

Source: The provisions of this §111.5 adopted to be effective September 10, 2012, 37 TexReg 7109; amended to be effective October 15, 2013, 38 TexReg 7112.

#### §111.6. Grade 4, Adopted 2012.

- (a) Introduction.
  - (1) The desire to achieve educational excellence is the driving force behind the Texas essential knowledge and skills for mathematics, guided by the college and career readiness standards. By embedding statistics, probability, and finance, while focusing on computational thinking, mathematical fluency, and solid understanding, Texas will lead the way in mathematics education and prepare all Texas students for the challenges they will face in the 21st century.
  - (2)The process standards describe ways in which students are expected to engage in the content. The placement of the process standards at the beginning of the knowledge and skills listed for each grade and course is intentional. The process standards weave the other knowledge and skills together so that students may be successful problem solvers and use mathematics efficiently and effectively in daily life. The process standards are integrated at every grade level and course. When possible, students will apply mathematics to problems arising in everyday life, society, and the workplace. Students will use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution. Students will select appropriate tools such as real objects, manipulatives, algorithms, paper and pencil, and technology and techniques such as mental math, estimation, number sense, and generalization and abstraction to solve problems. Students will effectively communicate mathematical ideas, reasoning, and their implications using multiple representations such as symbols, diagrams, graphs, computer programs, and language. Students will use mathematical relationships to generate solutions and make connections and predictions. Students will analyze mathematical relationships to connect and communicate mathematical ideas. Students will display, explain, or justify mathematical ideas and arguments using precise mathematical language in written or oral communication.
  - (3) For students to become fluent in mathematics, students must develop a robust sense of number. The National Research Council's report, "Adding It Up," defines procedural fluency as "skill in carrying out procedures flexibly, accurately, efficiently, and appropriately." As students develop procedural fluency, they must also realize that true problem solving may take time, effort, and perseverance. Students in Grade 4 are expected to perform their work without the use of calculators.
  - (4) The primary focal areas in Grade 4 are use of operations, fractions, and decimals and describing and analyzing geometry and measurement. These focal areas are supported throughout the mathematical strands of number and operations, algebraic reasoning, geometry and measurement, and data analysis. In Grades 3-5, the number set is limited to positive rational numbers. In number and operations, students will apply place value and represent points on a number line that correspond to a given fraction or terminating decimal. In algebraic reasoning, students will represent and solve multi-step problems involving the four operations with whole numbers with

expressions and equations and generate and analyze patterns. In geometry and measurement, students will classify two-dimensional figures, measure angles, and convert units of measure. In data analysis, students will represent and interpret data.

- (5) Statements that contain the word "including" reference content that must be mastered, while those containing the phrase "such as" are intended as possible illustrative examples.
- (b) Knowledge and skills.
  - (1) Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:
    - (A) apply mathematics to problems arising in everyday life, society, and the workplace;
    - (B) use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;
    - (C) select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;
    - (D) communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;
    - (E) create and use representations to organize, record, and communicate mathematical ideas;
    - (F) analyze mathematical relationships to connect and communicate mathematical ideas; and
    - (G) <u>display, explain, and justify\_mathematical ideas and arguments using precise</u> mathematical language in written or oral communication.
  - (2) Number and operations. The student applies mathematical process standards to represent, compare, and order whole numbers and decimals and understand relationships related to place value. The student is expected to:
    - (A) interpret the value of each place-value position as 10 times the position to the right and as one-tenth of the value of the place to its left;
    - (B) represent the value of the digit in whole numbers through 1,000,000,000 and decimals to the hundredths using expanded notation and numerals;
    - (C) compare and order whole numbers to 1,000,000,000 and represent comparisons using the symbols >, <, or =;
    - (D) round whole numbers to a given place value through the hundred thousands place;
    - (E) represent decimals, including tenths and hundredths, using concrete and visual models and money;
    - (F) compare and order decimals using concrete and visual models to the hundredths;
    - (G) relate decimals to fractions that name tenths and hundredths; and
    - (H) determine the corresponding decimal to the tenths or hundredths place of a specified point on a number line.
  - (3) Number and operations. The student applies mathematical process standards to represent and generate fractions to solve problems. The student is expected to:
    - (A) represent a fraction a/b as a sum of fractions 1/b, where a and b are whole numbers and b > 0, including when a > b;
    - (B) decompose a fraction in more than one way into a sum of fractions with the same denominator using concrete and pictorial models and recording results with symbolic representations;

- (C) determine if two given fractions are equivalent using a variety of methods;
- (D) compare two fractions with different numerators and different denominators and represent the comparison using the symbols >, =, or <;
- (E) represent and solve addition and subtraction of fractions with equal denominators using objects and pictorial models that build to the number line and properties of operations;
- (F) evaluate the reasonableness of sums and differences of fractions using benchmark fractions 0, 1/4, 1/2, 3/4, and 1, referring to the same whole; and
- (G) represent fractions and decimals to the tenths or hundredths as distances from zero on a number line.
- (4) Number and operations. The student applies mathematical process standards to develop and use strategies and methods for whole number computations and decimal sums and differences in order to solve problems with efficiency and accuracy. The student is expected to:
  - (A) add and subtract whole numbers and decimals to the hundredths place using the standard algorithm;
  - (B) determine products of a number and 10 or 100 using properties of operations and place value understandings;
  - (C) represent the product of 2 two-digit numbers using arrays, area models, or equations, including perfect squares through 15 by 15;
  - (D) use strategies and algorithms, including the standard algorithm, to multiply up to a fourdigit number by a one-digit number and to multiply a two-digit number by a two-digit number. Strategies may include mental math, partial products, and the commutative, associative, and distributive properties;
  - (E) represent the quotient of up to a four-digit whole number divided by a one-digit whole number using arrays, area models, or equations;
  - (F) use strategies and algorithms, including the standard algorithm, to divide up to a fourdigit dividend by a one-digit divisor;
  - (G) round to the nearest 10, 100, or 1,000 or use compatible numbers to estimate solutions involving whole numbers; and
  - (H) solve with fluency one- and two-step problems involving multiplication and division, including interpreting remainders.
- (5) Algebraic reasoning. The student applies mathematical process standards to develop concepts of expressions and equations. The student is expected to:
  - (A) represent multi-step problems involving the four operations with whole numbers using strip diagrams and equations with a letter standing for the unknown quantity;
  - (B) represent problems using an input-output table and numerical expressions to generate a number pattern that follows a given rule representing the relationship of the values in the resulting sequence and their position in the sequence;
  - (C) use models to determine the formulas for the perimeter of a rectangle (l + w + l + w or 2l + 2w), including the special form for perimeter of a square (4s) and the area of a rectangle  $(l \times w)$ ; and
  - (D) solve problems related to perimeter and area of rectangles where dimensions are whole numbers.
- (6) Geometry and measurement. The student applies mathematical process standards to analyze geometric attributes in order to develop generalizations about their properties. The student is expected to:

- (A) identify points, lines, line segments, rays, angles, and perpendicular and parallel lines;
- (B) identify and draw one or more lines of symmetry, if they exist, for a two-dimensional figure;
- (C) apply knowledge of right angles to identify acute, right, and obtuse triangles; and
- (D) classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines or the presence or absence of angles of a specified size.
- (7) Geometry and measurement. The student applies mathematical process standards to solve problems involving angles less than or equal to 180 degrees. The student is expected to:
  - (A) illustrate the measure of an angle as the part of a circle whose center is at the vertex of the angle that is "cut out" by the rays of the angle. Angle measures are limited to whole numbers;
  - (B) illustrate degrees as the units used to measure an angle, where 1/360 of any circle is one degree and an angle that "cuts" n/360 out of any circle whose center is at the angle's vertex has a measure of *n* degrees. Angle measures are limited to whole numbers;
  - (C) determine the approximate measures of angles in degrees to the nearest whole number using a protractor;
  - (D) draw an angle with a given measure; and
  - (E) determine the measure of an unknown angle formed by two non-overlapping adjacent angles given one or both angle measures.
- (8) Geometry and measurement. The student applies mathematical process standards to select appropriate customary and metric units, strategies, and tools to solve problems involving measurement. The student is expected to:
  - (A) identify relative sizes of measurement units within the customary and metric systems;
  - (B) convert measurements within the same measurement system, customary or metric, from a smaller unit into a larger unit or a larger unit into a smaller unit when given other equivalent measures represented in a table; and
  - (C) solve problems that deal with measurements of length, intervals of time, liquid volumes, mass, and money using addition, subtraction, multiplication, or division as appropriate.
- (9) Data analysis. The student applies mathematical process standards to solve problems by collecting, organizing, displaying, and interpreting data. The student is expected to:
  - (A) represent data on a frequency table, dot plot, or stem-and-leaf plot marked with whole numbers and fractions; and
  - (B) solve one- and two-step problems using data in whole number, decimal, and fraction form in a frequency table, dot plot, or stem-and-leaf plot.
- (10) Personal financial literacy. The student applies mathematical process standards to manage one's financial resources effectively for lifetime financial security. The student is expected to:
  - (A) distinguish between fixed and variable expenses;
  - (B) calculate profit in a given situation;
  - (C) compare the advantages and disadvantages of various savings options;
  - (D) describe how to allocate a weekly allowance among spending; saving, including for college; and sharing; and
  - (E) describe the basic purpose of financial institutions, including keeping money safe, borrowing money, and lending.

Source: The provisions of this §111.6 adopted to be effective September 10, 2012, 37 TexReg 7109.

## §111.7. Grade 5, Adopted 2012.

- (a) Introduction.
  - (1) The desire to achieve educational excellence is the driving force behind the Texas essential knowledge and skills for mathematics, guided by the college and career readiness standards. By embedding statistics, probability, and finance, while focusing on computational thinking, mathematical fluency, and solid understanding, Texas will lead the way in mathematics education and prepare all Texas students for the challenges they will face in the 21st century.
  - (2)The process standards describe ways in which students are expected to engage in the content. The placement of the process standards at the beginning of the knowledge and skills listed for each grade and course is intentional. The process standards weave the other knowledge and skills together so that students may be successful problem solvers and use mathematics efficiently and effectively in daily life. The process standards are integrated at every grade level and course. When possible, students will apply mathematics to problems arising in everyday life, society, and the workplace. Students will use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution. Students will select appropriate tools such as real objects, manipulatives, algorithms, paper and pencil, and technology and techniques such as mental math, estimation, number sense, and generalization and abstraction to solve problems. Students will effectively communicate mathematical ideas, reasoning, and their implications using multiple representations such as symbols, diagrams, graphs, computer programs, and language. Students will use mathematical relationships to generate solutions and make connections and predictions. Students will analyze mathematical relationships to connect and communicate mathematical ideas. Students will display, explain, or justify mathematical ideas and arguments using precise mathematical language in written or oral communication.
  - (3) For students to become fluent in mathematics, students must develop a robust sense of number. The National Research Council's report, "Adding It Up," defines procedural fluency as "skill in carrying out procedures flexibly, accurately, efficiently, and appropriately." As students develop procedural fluency, they must also realize that true problem solving may take time, effort, and perseverance. Students in Grade 5 are expected to perform their work without the use of calculators.
  - (4) The primary focal areas in Grade 5 are solving problems involving all four operations with positive rational numbers, determining and generating formulas and solutions to expressions, and extending measurement to area and volume. These focal areas are supported throughout the mathematical strands of number and operations, algebraic reasoning, geometry and measurement, and data analysis. In Grades 3-5, the number set is limited to positive rational numbers. In number and operations, students will apply place value and identify part-to-whole relationships and equivalence. In algebraic reasoning, students will represent and solve problems with expressions and equations, build foundations of functions through patterning, identify prime and composite numbers, and use the order of operations. In geometry and measurement, students will classify two-dimensional figures, connect geometric attributes to the measures of three-dimensional figures, use units of measure, and represent location using a coordinate plane. In data analysis, students will represent and interpret data.
  - (5) Statements that contain the word "including" reference content that must be mastered, while those containing the phrase "such as" are intended as possible illustrative examples.
- (b) Knowledge and skills.
  - (1) Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:
    - (A) apply mathematics to problems arising in everyday life, society, and the workplace;

- (B) use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;
- select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;
- (D) communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;
- (E) create and use representations to organize, record, and communicate mathematical ideas;
- (F) analyze mathematical relationships to connect and communicate mathematical ideas; and
- (G) display, explain, and justify\_mathematical ideas and arguments using precise mathematical language in written or oral communication.
- (2) Number and operations. The student applies mathematical process standards to represent, compare, and order positive rational numbers and understand relationships as related to place value. The student is expected to:
  - (A) represent the value of the digit in decimals through the thousandths using expanded notation and numerals;
  - (B) compare and order two decimals to thousandths and represent comparisons using the symbols >, <, or =; and
  - (C) round decimals to tenths or hundredths.
- (3) Number and operations. The student applies mathematical process standards to develop and use strategies and methods for positive rational number computations in order to solve problems with efficiency and accuracy. The student is expected to:
  - (A) estimate to determine solutions to mathematical and real-world problems involving addition, subtraction, multiplication, or division;
  - (B) multiply with fluency a three-digit number by a two-digit number using the standard algorithm;
  - (C) solve with proficiency for quotients of up to a four-digit dividend by a two-digit divisor using strategies and the standard algorithm;
  - (D) represent multiplication of decimals with products to the hundredths using objects and pictorial models, including area models;
  - (E) solve for products of decimals to the hundredths, including situations involving money, using strategies based on place-value understandings, properties of operations, and the relationship to the multiplication of whole numbers;
  - (F) represent quotients of decimals to the hundredths, up to four-digit dividends and twodigit whole number divisors, using objects and pictorial models, including area models;
  - (G) solve for quotients of decimals to the hundredths, up to four-digit dividends and two-digit whole number divisors, using strategies and algorithms, including the standard algorithm;
  - (H) represent and solve addition and subtraction of fractions with unequal denominators referring to the same whole using objects and pictorial models and properties of operations;
  - (I) represent and solve multiplication of a whole number and a fraction that refers to the same whole using objects and pictorial models, including area models;

- (J) represent division of a unit fraction by a whole number and the division of a whole number by a unit fraction such as  $1/3 \div 7$  and  $7 \div 1/3$  using objects and pictorial models, including area models;
- (K) add and subtract positive rational numbers fluently; and
- (L) divide whole numbers by unit fractions and unit fractions by whole numbers.
- (4) Algebraic reasoning. The student applies mathematical process standards to develop concepts of expressions and equations. The student is expected to:
  - (A) identify prime and composite numbers;
  - (B) represent and solve multi-step problems involving the four operations with whole numbers using equations with a letter standing for the unknown quantity;
  - (C) generate a numerical pattern when given a rule in the form y = ax or y = x + a and graph;
  - (D) recognize the difference between additive and multiplicative numerical patterns given in a table or graph;
  - (E) describe the meaning of parentheses and brackets in a numeric expression;
  - (F) simplify numerical expressions that do not involve exponents, including up to two levels of grouping;
  - (G) use concrete objects and pictorial models to develop the formulas for the volume of a rectangular prism, including the special form for a cube  $(V = l \ge w \ge h, V = s \ge s \ge s, s \ge s, s \ge b)$ ; and
  - (H) represent and solve problems related to perimeter and/or area and related to volume.
- (5) Geometry and measurement. The student applies mathematical process standards to classify twodimensional figures by attributes and properties. The student is expected to classify twodimensional figures in a hierarchy of sets and subsets using graphic organizers based on their attributes and properties.
- (6) Geometry and measurement. The student applies mathematical process standards to understand, recognize, and quantify volume. The student is expected to:
  - (A) recognize a cube with side length of one unit as a unit cube having one cubic unit of volume and the volume of a three-dimensional figure as the number of unit cubes (*n* cubic units) needed to fill it with no gaps or overlaps if possible; and
  - (B) determine the volume of a rectangular prism with whole number side lengths in problems related to the number of layers times the number of unit cubes in the area of the base.
- (7) Geometry and measurement. The student applies mathematical process standards to select appropriate units, strategies, and tools to solve problems involving measurement. The student is expected to solve problems by calculating conversions within a measurement system, customary or metric.
- (8) Geometry and measurement. The student applies mathematical process standards to identify locations on a coordinate plane. The student is expected to:
  - (A) describe the key attributes of the coordinate plane, including perpendicular number lines (axes) where the intersection (origin) of the two lines coincides with zero on each number line and the given point (0, 0); the *x*-coordinate, the first number in an ordered pair, indicates movement parallel to the *x*-axis starting at the origin; and the *y*-coordinate, the second number, indicates movement parallel to the *y*-axis starting at the origin;
  - (B) describe the process for graphing ordered pairs of numbers in the first guadrant of the coordinate plane; and

- (C) graph in the first quadrant of the coordinate plane ordered pairs of numbers arising from mathematical and real-world problems, including those generated by number patterns or found in an input-output table.
- (9) Data analysis. The student applies mathematical process standards to solve problems by collecting, organizing, displaying, and interpreting data. The student is expected to:
  - (A) represent categorical data with bar graphs or frequency tables and numerical data, including data sets of measurements in fractions or decimals, with dot plots or stem-andleaf plots;
  - (B) represent discrete paired data on a scatterplot; and
  - (C) solve one- and two-step problems using data from a frequency table, dot plot, bar graph, stem-and-leaf plot, or scatterplot.
- (10) Personal financial literacy. The student applies mathematical process standards to manage one's financial resources effectively for lifetime financial security. The student is expected to:
  - (A) define income tax, payroll tax, sales tax, and property tax;
  - (B) explain the difference between gross income and net income;
  - (C) identify the advantages and disadvantages of different methods of payment, including check, credit card, debit card, and electronic payments;
  - (D) develop a system for keeping and using financial records;
  - (E) describe actions that might be taken to balance a budget when expenses exceed income; and
  - (F) balance a simple budget.

Source: The provisions of this §111.7 adopted to be effective September 10, 2012, 37 TexReg 7109.

## §111.11. Implementation of Texas Essential Knowledge and Skills for Mathematics, Grades K-5.

The provisions of this subchapter shall be implemented by school districts beginning with the 2006-2007 school year.

Source: The provisions of this §111.11 adopted to be effective September 1, 1998, 22 TexReg 7623; amended to be effective August 1, 2006, 30 TexReg 7471.

## §111.12. Mathematics, Kindergarten.

- (a) Introduction.
  - (1) Within a well-balanced mathematics curriculum, the primary focal points at Kindergarten are developing whole-number concepts and using patterns and sorting to explore number, data, and shape.
  - (2) Throughout mathematics in Kindergarten-Grade 2, students build a foundation of basic understandings in number, operation, and quantitative reasoning; patterns, relationships, and algebraic thinking; geometry and spatial reasoning; measurement; and probability and statistics. Students use numbers in ordering, labeling, and expressing quantities and relationships to solve problems and translate informal language into mathematical language and symbols. Students use objects to create and identify patterns and use those patterns to express relationships, make predictions, and solve problems as they build an understanding of number, operation, shape, and space. Students progress from informal to formal language to describe two- and three-dimensional geometric figures and likenesses in the physical world. Students begin to develop measurement concepts as they identify and compare attributes of objects and situations. Students collect, organize, and display data and use information from graphs to answer questions, make summary statements, and make informal predictions based on their experiences.

- (3) Throughout mathematics in Kindergarten-Grade 2, students develop numerical fluency with conceptual understanding and computational accuracy. Students in Kindergarten-Grade 2 use basic number sense to compose and decompose numbers in order to solve problems requiring precision, estimation, and reasonableness. By the end of Grade 2, students know basic addition and subtraction facts and are using them to work flexibly, efficiently, and accurately with numbers during addition and subtraction computation.
- (4) Problem solving, language and communication, connections within and outside mathematics, and formal and informal reasoning underlie all content areas in mathematics. Throughout mathematics in Kindergarten-Grade 2, students use these processes together with technology and other mathematical tools such as manipulative materials to develop conceptual understanding and solve meaningful problems as they do mathematics.
- (b) Knowledge and skills.
  - (1) Number, operation, and quantitative reasoning. The student uses numbers to name quantities. The student is expected to:
    - (A) use one-to-one correspondence and language such as more than, same number as, or two less than to describe relative sizes of sets of concrete objects;
    - (B) use sets of concrete objects to represent quantities given in verbal or written form (through 20); and
    - (C) use numbers to describe how many objects are in a set (through 20) using verbal and symbolic descriptions.
  - (2) Number, operation, and quantitative reasoning. The student describes order of events or objects. The student is expected to:
    - (A) use language such as before or after to describe relative position in a sequence of events or objects; and
    - (B) name the ordinal positions in a sequence such as first, second, third, etc.
  - (3) Number, operation, and quantitative reasoning. The student recognizes that there are quantities less than a whole. The student is expected to:
    - (A) share a whole by separating it into two equal parts; and
    - (B) explain why a given part is half of the whole.
  - (4) Number, operation, and quantitative reasoning. The student models addition (joining) and subtraction (separating). The student is expected to model and create addition and subtraction problems in real situations with concrete objects.
  - (5) Patterns, relationships, and algebraic thinking. The student identifies, extends, and creates patterns. The student is expected to identify, extend, and create patterns of sounds, physical movement, and concrete objects.
  - (6) Patterns, relationships, and algebraic thinking. The student uses patterns to make predictions. The student is expected to:
    - (A) use patterns to predict what comes next, including cause-and-effect relationships; and
    - (B) count by ones to 100.
  - (7) Geometry and spatial reasoning. The student describes the relative positions of objects. The student is expected to:
    - (A) describe one object in relation to another using informal language such as over, under, above, and below; and
    - (B) place an object in a specified position.

- (8) Geometry and spatial reasoning. The student uses attributes to determine how objects are alike and different. The student is expected to:
  - (A) describe and identify an object by its attributes using informal language;
  - (B) compare two objects based on their attributes; and
  - (C) sort a variety of objects including two- and three-dimensional geometric figures according to their attributes and describe how the objects are sorted.
- (9) Geometry and spatial reasoning. The student recognizes attributes of two- and three-dimensional geometric figures. The student is expected to:
  - (A) describe and compare the attributes of real-life objects such as balls, boxes, cans, and cones or models of three-dimensional geometric figures;
  - (B) recognize shapes in real-life three-dimensional geometric figures or models of threedimensional geometric figures; and
  - (C) describe, identify, and compare circles, triangles, rectangles, and squares (a special type of rectangle).
- (10) Measurement. The student directly compares the attributes of length, area, weight/mass, capacity, and/or relative temperature. The student uses comparative language to solve problems and answer questions. The student is expected to:
  - (A) compare and order two or three concrete objects according to length (longer/shorter than, or the same);
  - (B) compare the areas of two flat surfaces of two-dimensional figures (covers more, covers less, or covers the same);
  - (C) compare two containers according to capacity (holds more, holds less, or holds the same);
  - (D) compare two objects according to weight/mass (heavier than, lighter than or equal to); and
  - (E) compare situations or objects according to relative temperature (hotter/colder than, or the same as).
- (11) Measurement. The student uses time to describe, compare, and order events and situations. The student is expected to:
  - (A) compare events according to duration such as more time than or less time than;
  - (B) sequence events (up to three); and
  - (C) read a calendar using days, weeks, and months.
- (12) Probability and statistics. The student constructs and uses graphs of real objects or pictures to answer questions. The student is expected to:
  - (A) construct graphs using real objects or pictures in order to answer questions; and
  - (B) use information from a graph of real objects or pictures in order to answer questions.
- (13) Underlying processes and mathematical tools. The student applies Kindergarten mathematics to solve problems connected to everyday experiences and activities in and outside of school. The student is expected to:
  - (A) identify mathematics in everyday situations;
  - (B) solve problems with guidance that incorporates the processes of understanding the problem, making a plan, carrying out the plan, and evaluating the solution for reasonableness;

- (C) select or develop an appropriate problem-solving strategy including drawing a picture, looking for a pattern, systematic guessing and checking, or acting it out in order to solve a problem; and
- (D) use tools such as real objects, manipulatives, and technology to solve problems.
- (14) Underlying processes and mathematical tools. The student communicates about Kindergarten mathematics using informal language. The student is expected to:
  - (A) communicate mathematical ideas using objects, words, pictures, numbers, and technology; and
  - (B) relate everyday language to mathematical language and symbols.
- (15) Underlying processes and mathematical tools. The student uses logical reasoning. The student is expected to justify his or her thinking using objects, words, pictures, numbers, and technology.

Source: The provisions of this §111.12 adopted to be effective September 1, 1998, 22 TexReg 7623; amended to be effective August 1, 2006, 30 TexReg 7471.

## §111.13. Mathematics, Grade 1.

- (a) Introduction.
  - (1) Within a well-balanced mathematics curriculum, the primary focal points at Grade 1 are building number sense through number relationships, adding and subtracting whole numbers, organizing and analyzing data, and working with two- and three-dimensional geometric figures.
  - (2) Throughout mathematics in Kindergarten-Grade 2, students build a foundation of basic understandings in number, operation, and quantitative reasoning; patterns, relationships, and algebraic thinking; geometry and spatial reasoning; measurement; and probability and statistics. Students use numbers in ordering, labeling, and expressing quantities and relationships to solve problems and translate informal language into mathematical language and symbols. Students use objects to create and identify patterns and use those patterns to express relationships, make predictions, and solve problems as they build an understanding of number, operation, shape, and space. Students progress from informal to formal language to describe two- and three-dimensional geometric figures and likenesses in the physical world. Students begin to develop measurement concepts as they identify and compare attributes of objects and situations. Students collect, organize, and display data and use information from graphs to answer questions, make summary statements, and make informal predictions based on their experiences.
  - (3) Throughout mathematics in Kindergarten-Grade 2, students develop numerical fluency with conceptual understanding and computational accuracy. Students in Kindergarten-Grade 2 use basic number sense to compose and decompose numbers in order to solve problems requiring precision, estimation, and reasonableness. By the end of Grade 2, students know basic addition and subtraction facts and are using them to work flexibly, efficiently, and accurately with numbers during addition and subtraction computation.
  - (4) Problem solving, language and communication, connections within and outside mathematics, and formal and informal reasoning underlie all content areas in mathematics. Throughout mathematics in Kindergarten-Grade 2, students use these processes together with technology and other mathematical tools such as manipulative materials to develop conceptual understanding and solve meaningful problems as they do mathematics.
- (b) Knowledge and skills.
  - (1) Number, operation, and quantitative reasoning. The student uses whole numbers to describe and compare quantities. The student is expected to:
    - (A) compare and order whole numbers up to 99 (less than, greater than, or equal to) using sets of concrete objects and pictorial models;

- (B) create sets of tens and ones using concrete objects to describe, compare, and order whole numbers;
- (C) identify individual coins by name and value and describe relationships among them; and
- (D) read and write numbers to 99 to describe sets of concrete objects.
- (2) Number, operation, and quantitative reasoning. The student uses pairs of whole numbers to describe fractional parts of whole objects or sets of objects. The student is expected to:
  - (A) separate a whole into two, three, or four equal parts and use appropriate language to describe the parts such as three out of four equal parts; and
  - (B) use appropriate language to describe part of a set such as three out of the eight crayons are red.
- (3) Number, operation, and quantitative reasoning. The student recognizes and solves problems in addition and subtraction situations. The student is expected to:
  - (A) model and create addition and subtraction problem situations with concrete objects and write corresponding number sentences; and
  - (B) use concrete and pictorial models to apply basic addition and subtraction facts (up to 9 + 9 = 18 and 18 9 = 9).
- (4) Patterns, relationships, and algebraic thinking. The student uses repeating patterns and additive patterns to make predictions. The student is expected to identify, describe, and extend concrete and pictorial patterns in order to make predictions and solve problems.
- (5) Patterns, relationships, and algebraic thinking. The student recognizes patterns in numbers and operations. The student is expected to:
  - (A) use patterns to skip count by twos, fives, and tens;
  - (B) find patterns in numbers, including odd and even;
  - (C) compare and order whole numbers using place value;
  - (D) use patterns to develop strategies to solve basic addition and basic subtraction problems; and
  - (E) identify patterns in related addition and subtraction sentences (fact families for sums to 18) such as 2 + 3 = 5, 3 + 2 = 5, 5 2 = 3, and 5 3 = 2.
- (6) Geometry and spatial reasoning. The student uses attributes to identify two- and three-dimensional geometric figures. The student compares and contrasts two- and three-dimensional geometric figures or both. The student is expected to:
  - (A) describe and identify two-dimensional geometric figures, including circles, triangles, rectangles, and squares (a special type of rectangle);
  - (B) describe and identify three-dimensional geometric figures, including spheres, rectangular prisms (including cubes), cylinders, and cones;
  - (C) describe and identify two- and three-dimensional geometric figures in order to sort them according to a given attribute using informal and formal language; and
  - (D) use concrete models to combine two-dimensional geometric figures to make new geometric figures.
- Measurement. The student directly compares the attributes of length, area, weight/mass, capacity, and temperature. The student uses comparative language to solve problems and answer questions. The student selects and uses nonstandard units to describe length. The student is expected to:
  - (A) estimate and measure length using nonstandard units such as paper clips or sides of color tiles;

- (B) compare and order two or more concrete objects according to length (from longest to shortest);
- (C) describe the relationship between the size of the unit and the number of units needed to measure the length of an object;
- (D) compare and order the area of two or more two-dimensional surfaces (from covers the most to covers the least);
- (E) compare and order two or more containers according to capacity (from holds the most to holds the least);
- (F) compare and order two or more objects according to weight/mass (from heaviest to lightest); and
- (G) compare and order two or more objects according to relative temperature (from hottest to coldest).
- (8) Measurement. The student understands that time can be measured. The student uses time to describe and compare situations. The student is expected to:
  - (A) order three or more events according to duration; and
  - (B) read time to the hour and half-hour using analog and digital clocks.
- (9) Probability and statistics. The student displays data in an organized form. The student is expected to:
  - (A) collect and sort data; and
  - (B) use organized data to construct real-object graphs, picture graphs, and bar-type graphs.
- (10) Probability and statistics. The student uses information from organized data. The student is expected to:
  - (A) draw conclusions and answer questions using information organized in real-object graphs, picture graphs, and bar-type graphs; and
  - (B) identify events as certain or impossible such as drawing a red crayon from a bag of green crayons.
- (11) Underlying processes and mathematical tools. The student applies Grade 1 mathematics to solve problems connected to everyday experiences and activities in and outside of school. The student is expected to:
  - (A) identify mathematics in everyday situations;
  - (B) solve problems with guidance that incorporates the processes of understanding the problem, making a plan, carrying out the plan, and evaluating the solution for reasonableness;
  - (C) select or develop an appropriate problem-solving plan or strategy including drawing a picture, looking for a pattern, systematic guessing and checking, or acting it out in order to solve a problem; and
  - (D) use tools such as real objects, manipulatives, and technology to solve problems.
- (12) Underlying processes and mathematical tools. The student communicates about Grade 1 mathematics using informal language. The student is expected to:
  - (A) explain and record observations using objects, words, pictures, numbers, and technology; and
  - (B) relate informal language to mathematical language and symbols.
- (13) Underlying processes and mathematical tools. The student uses logical reasoning. The student is expected to justify his or her thinking using objects, words, pictures, numbers, and technology.

Source: The provisions of this §111.13 adopted to be effective September 1, 1998, 22 TexReg 7623; amended to be effective August 1, 2006, 30 TexReg 7471.

## §111.14. Mathematics, Grade 2.

- (a) Introduction.
  - (1) Within a well-balanced mathematics curriculum, the primary focal points at Grade 2 are developing an understanding of the base-ten place value system, comparing and ordering whole numbers, applying addition and subtraction, and using measurement processes.
  - (2) Throughout mathematics in Kindergarten-Grade 2, students build a foundation of basic understandings in number, operation, and quantitative reasoning; patterns, relationships, and algebraic thinking; geometry and spatial reasoning; measurement; and probability and statistics. Students use numbers in ordering, labeling, and expressing quantities and relationships to solve problems and translate informal language into mathematical language and symbols. Students use objects to create and identify patterns and use those patterns to express relationships, make predictions, and solve problems as they build an understanding of number, operation, shape, and space. Students progress from informal to formal language to describe two- and three-dimensional geometric figures and likenesses in the physical world. Students begin to develop measurement concepts as they identify and compare attributes of objects and situations. Students collect, organize, and display data and use information from graphs to answer questions, make summary statements, and make informal predictions based on their experiences.
  - (3) Throughout mathematics in Kindergarten-Grade 2, students develop numerical fluency with conceptual understanding and computational accuracy. Students in Kindergarten-Grade 2 use basic number sense to compose and decompose numbers in order to solve problems requiring precision, estimation, and reasonableness. By the end of Grade 2, students know basic addition and subtraction facts and are using them to work flexibly, efficiently, and accurately with numbers during addition and subtraction computation.
  - (4) Problem solving, language and communication, connections within and outside mathematics, and formal and informal reasoning underlie all content areas in mathematics. Throughout mathematics in Kindergarten-Grade 2, students use these processes together with technology and other mathematical tools such as manipulative materials to develop conceptual understanding and solve meaningful problems as they do mathematics.
- (b) Knowledge and skills.
  - (1) Number, operation, and quantitative reasoning. The student understands how place value is used to represent whole numbers. The student is expected to:
    - (A) use concrete models of hundreds, tens, and ones to represent a given whole number (up to 999) in various ways;
    - (B) use place value to read, write, and describe the value of whole numbers to 999; and
    - (C) use place value to compare and order whole numbers to 999 and record the comparisons using numbers and symbols (<, =, >).
  - (2) Number, operation, and quantitative reasoning. The student describes how fractions are used to name parts of whole objects or sets of objects. The student is expected to:
    - (A) use concrete models to represent and name fractional parts of a whole object (with denominators of 12 or less);
    - (B) use concrete models to represent and name fractional parts of a set of objects (with denominators of 12 or less); and
    - (C) use concrete models to determine if a fractional part of a whole is closer to 0, ½, or 1.
  - (3) Number, operation, and quantitative reasoning. The student adds and subtracts whole numbers to solve problems. The student is expected to:

- (A) recall and apply basic addition and subtraction facts ( to 18);
- (B) model addition and subtraction of two-digit numbers with objects, pictures, words, and numbers;
- (C) select addition or subtraction to solve problems using two-digit numbers, whether or not regrouping is necessary;
- (D) determine the value of a collection of coins up to one dollar; and
- (E) describe how the cent symbol, dollar symbol, and the decimal point are used to name the value of a collection of coins.
- (4) Number, operation, and quantitative reasoning. The student models multiplication and division. The student is expected to:
  - (A) model, create, and describe multiplication situations in which equivalent sets of concrete objects are joined; and
  - (B) model, create, and describe division situations in which a set of concrete objects is separated into equivalent sets.
- (5) Patterns, relationships, and algebraic thinking. The student uses patterns in numbers and operations. The student is expected to:
  - (A) find patterns in numbers such as in a 100s chart;
  - (B) use patterns in place value to compare and order whole numbers through 999; and
  - (C) use patterns and relationships to develop strategies to remember basic addition and subtraction facts. Determine patterns in related addition and subtraction number sentences (including fact families) such as 8 + 9 = 17, 9 + 8 = 17, 17 8 = 9, and 17 9 = 8.
- (6) Patterns, relationships, and algebraic thinking. The student uses patterns to describe relationships and make predictions. The student is expected to:
  - (A) generate a list of paired numbers based on a real-life situation such as number of tricycles related to number of wheels;
  - (B) identify patterns in a list of related number pairs based on a real-life situation and extend the list; and
  - (C) identify, describe, and extend repeating and additive patterns to make predictions and solve problems.
- (7) Geometry and spatial reasoning. The student uses attributes to identify two- and three-dimensional geometric figures. The student compares and contrasts two- and three-dimensional geometric figures or both. The student is expected to:
  - (A) describe attributes (the number of vertices, faces, edges, sides) of two- and threedimensional geometric figures such as circles, polygons, spheres, cones, cylinders, prisms, and pyramids, etc.;
  - (B) use attributes to describe how 2 two-dimensional figures or 2 three-dimensional geometric figures are alike or different; and
  - (C) cut two-dimensional geometric figures apart and identify the new geometric figures formed.
- (8) Geometry and spatial reasoning. The student recognizes that a line can be used to represent a set of numbers and its properties. The student is expected to use whole numbers to locate and name points on a number line.
- (9) Measurement. The student directly compares the attributes of length, area, weight/mass, and capacity, and uses comparative language to solve problems and answer questions. The student

selects and uses nonstandard units to describe length, area, capacity, and weight/mass. The student recognizes and uses models that approximate standard units (from both SI, also known as metric, and customary systems) of length, weight/mass, capacity, and time. The student is expected to:

- (A) identify concrete models that approximate standard units of length and use them to measure length;
- (B) select a non-standard unit of measure such as square tiles to determine the area of a twodimensional surface;
- (C) select a non-standard unit of measure such as a bathroom cup or a jar to determine the capacity of a given container; and
- (D) select a non-standard unit of measure such as beans or marbles to determine the weight/mass of a given object.
- (10) Measurement. The student uses standard tools to estimate and measure time and temperature (in degrees Fahrenheit). The student is expected to:
  - (A) read a thermometer to gather data;
  - (B) read and write times shown on analog and digital clocks using five-minute increments; and
  - (C) describe activities that take approximately one second, one minute, and one hour.
- (11) Probability and statistics. The student organizes data to make it useful for interpreting information. The student is expected to:
  - (A) construct picture graphs and bar-type graphs;
  - (B) draw conclusions and answer questions based on picture graphs and bar-type graphs; and
  - (C) use data to describe events as more likely or less likely such as drawing a certain color crayon from a bag of seven red crayons and three green crayons.
- (12) Underlying processes and mathematical tools. The student applies Grade 2 mathematics to solve problems connected to everyday experiences and activities in and outside of school. The student is expected to:
  - (A) identify the mathematics in everyday situations;
  - (B) solve problems with guidance that incorporates the processes of understanding the problem, making a plan, carrying out the plan, and evaluating the solution for reasonableness;
  - (C) select or develop an appropriate problem-solving plan or strategy including drawing a picture, looking for a pattern, systematic guessing and checking, or acting it out in order to solve a problem; and
  - (D) use tools such as real objects, manipulatives, and technology to solve problems.
- (13) Underlying processes and mathematical tools. The student communicates about Grade 2 mathematics using informal language. The student is expected to:
  - (A) explain and record observations using objects, words, pictures, numbers, and technology; and
  - (B) relate informal language to mathematical language and symbols.
- (14) Underlying processes and mathematical tools. The student uses logical reasoning. The student is expected to justify his or her thinking using objects, words, pictures, numbers, and technology.

Source: The provisions of this §111.14 adopted to be effective September 1, 1998, 22 TexReg 7623; amended to be effective August 1, 2006, 30 TexReg 7471.

## §111.15. Mathematics, Grade 3.

- (a) Introduction.
  - (1) Within a well-balanced mathematics curriculum, the primary focal points at Grade 3 are multiplying and dividing whole numbers, connecting fraction symbols to fractional quantities, and standardizing language and procedures in geometry and measurement.
  - (2) Throughout mathematics in Grades 3-5, students build a foundation of basic understandings in number, operation, and quantitative reasoning; patterns, relationships, and algebraic thinking; geometry and spatial reasoning; measurement; and probability and statistics. Students use algorithms for addition, subtraction, multiplication, and division as generalizations connected to concrete experiences; and they concretely develop basic concepts of fractions and decimals. Students use appropriate language and organizational structures such as tables and charts to represent and communicate relationships, make predictions, and solve problems. Students select and use formal language to describe their reasoning as they identify, compare, and classify two- or three-dimensional geometric figures; and they use numbers, standard units, and measurement tools to describe and compare objects, make estimates, and solve application problems. Students organize data, choose an appropriate method to display the data, and interpret the data to make decisions and predictions and solve problems.
  - (3) Throughout mathematics in Grades 3-5, students develop numerical fluency with conceptual understanding and computational accuracy. Students in Grades 3-5 use knowledge of the base-ten place value system to compose and decompose numbers in order to solve problems requiring precision, estimation, and reasonableness. By the end of Grade 5, students know basic addition, subtraction, multiplication, and division facts and are using them to work flexibly, efficiently, and accurately with numbers during addition, subtraction, multiplication, and division computation.
  - (4) Problem solving, language and communication, connections within and outside mathematics, and formal and informal reasoning underlie all content areas in mathematics. Throughout mathematics in Grades 3-5, students use these processes together with technology and other mathematical tools such as manipulative materials to develop conceptual understanding and solve meaningful problems as they do mathematics.
- (b) Knowledge and skills.
  - (1) Number, operation, and quantitative reasoning. The student uses place value to communicate about increasingly large whole numbers in verbal and written form, including money. The student is expected to:
    - (A) use place value to read, write (in symbols and words), and describe the value of whole numbers through 999,999;
    - (B) use place value to compare and order whole numbers through 9,999; and
    - (C) determine the value of a collection of coins and bills.
  - (2) Number, operation, and quantitative reasoning. The student uses fraction names and symbols (with denominators of 12 or less) to describe fractional parts of whole objects or sets of objects. The student is expected to:
    - (A) construct concrete models of fractions;
    - (B) compare fractional parts of whole objects or sets of objects in a problem situation using concrete models;
    - (C) use fraction names and symbols to describe fractional parts of whole objects or sets of objects; and
    - (D) construct concrete models of equivalent fractions for fractional parts of whole objects.
  - (3) Number, operation, and quantitative reasoning. The student adds and subtracts to solve meaningful problems involving whole numbers. The student is expected to:

- (A) model addition and subtraction using pictures, words, and numbers; and
- (B) select addition or subtraction and use the operation to solve problems involving whole numbers through 999.
- (4) Number, operation, and quantitative reasoning. The student recognizes and solves problems in multiplication and division situations. The student is expected to:
  - (A) learn and apply multiplication facts through 12 by 12 using concrete models and objects;
  - (B) solve and record multiplication problems (up to two digits times one digit); and
  - (C) use models to solve division problems and use number sentences to record the solutions.
- (5) Number, operation, and quantitative reasoning. The student estimates to determine reasonable results. The student is expected to:
  - (A) round whole numbers to the nearest ten or hundred to approximate reasonable results in problem situations; and
  - (B) use strategies including rounding and compatible numbers to estimate solutions to addition and subtraction problems.
- (6) Patterns, relationships, and algebraic thinking. The student uses patterns to solve problems. The student is expected to:
  - (A) identify and extend whole-number and geometric patterns to make predictions and solve problems;
  - (B) identify patterns in multiplication facts using concrete objects, pictorial models, or technology; and
  - (C) identify patterns in related multiplication and division sentences (fact families) such as 2 x = 3 = 6, 3 = 2, 3 = 2, 6 = 3, 6 = 2.
- (7) Patterns, relationships, and algebraic thinking. The student uses lists, tables, and charts to express patterns and relationships. The student is expected to:
  - (A) generate a table of paired numbers based on a real-life situation such as insects and legs; and
  - (B) identify and describe patterns in a table of related number pairs based on a meaningful problem and extend the table.
- (8) Geometry and spatial reasoning. The student uses formal geometric vocabulary. The student is expected to identify, classify, and describe two- and three-dimensional geometric figures by their attributes. The student compares two- dimensional figures, three-dimensional figures, or both by their attributes using formal geometry vocabulary.
- (9) Geometry and spatial reasoning. The student recognizes congruence and symmetry. The student is expected to:
  - (A) identify congruent two-dimensional figures;
  - (B) create two-dimensional figures with lines of symmetry using concrete models and technology; and
  - (C) identify lines of symmetry in two-dimensional geometric figures.
- (10) Geometry and spatial reasoning. The student recognizes that a line can be used to represent numbers and fractions and their properties and relationships. The student is expected to locate and name points on a number line using whole numbers and fractions, including halves and fourths.
- (11) Measurement. The student directly compares the attributes of length, area, weight/mass, and capacity, and uses comparative language to solve problems and answer questions. The student

selects and uses standard units to describe length, area, capacity/volume, and weight/mass. The student is expected to:

- (A) use linear measurement tools to estimate and measure lengths using standard units;
- (B) use standard units to find the perimeter of a shape;
- (C) use concrete and pictorial models of square units to determine the area of twodimensional surfaces;
- (D) identify concrete models that approximate standard units of weight/mass and use them to measure weight/mass;
- (E) identify concrete models that approximate standard units for capacity and use them to measure capacity; and
- (F) use concrete models that approximate cubic units to determine the volume of a given container or other three-dimensional geometric figure.
- (12) Measurement. The student reads and writes time and measures temperature in degrees Fahrenheit to solve problems. The student is expected to:
  - (A) use a thermometer to measure temperature; and
  - (B) tell and write time shown on analog and digital clocks.
- (13) Probability and statistics. The student solves problems by collecting, organizing, displaying, and interpreting sets of data. The student is expected to:
  - (A) collect, organize, record, and display data in pictographs and bar graphs where each picture or cell might represent more than one piece of data;
  - (B) interpret information from pictographs and bar graphs; and
  - (C) use data to describe events as more likely than, less likely than, or equally likely as.
- (14) Underlying processes and mathematical tools. The student applies Grade 3 mathematics to solve problems connected to everyday experiences and activities in and outside of school. The student is expected to:
  - (A) identify the mathematics in everyday situations;
  - (B) solve problems that incorporate understanding the problem, making a plan, carrying out the plan, and evaluating the solution for reasonableness;
  - (C) select or develop an appropriate problem-solving plan or strategy, including drawing a picture, looking for a pattern, systematic guessing and checking, acting it out, making a table, working a simpler problem, or working backwards to solve a problem; and
  - (D) use tools such as real objects, manipulatives, and technology to solve problems.
- (15) Underlying processes and mathematical tools. The student communicates about Grade 3 mathematics using informal language. The student is expected to:
  - (A) explain and record observations using objects, words, pictures, numbers, and technology; and
  - (B) relate informal language to mathematical language and symbols.
- (16) Underlying processes and mathematical tools. The student uses logical reasoning. The student is expected to:
  - (A) make generalizations from patterns or sets of examples and nonexamples; and
  - (B) justify why an answer is reasonable and explain the solution process.

Source: The provisions of this §111.15 adopted to be effective September 1, 1998, 22 TexReg 7623; amended to be effective August 1, 2006, 30 TexReg 7471.

## §111.16. Mathematics, Grade 4.

- (a) Introduction.
  - (1) Within a well-balanced mathematics curriculum, the primary focal points at Grade 4 are comparing and ordering fractions and decimals, applying multiplication and division, and developing ideas related to congruence and symmetry.
  - (2) Throughout mathematics in Grades 3-5, students build a foundation of basic understandings in number, operation, and quantitative reasoning; patterns, relationships, and algebraic thinking; geometry and spatial reasoning; measurement; and probability and statistics. Students use algorithms for addition, subtraction, multiplication, and division as generalizations connected to concrete experiences; and they concretely develop basic concepts of fractions and decimals. Students use appropriate language and organizational structures such as tables and charts to represent and communicate relationships, make predictions, and solve problems. Students select and use formal language to describe their reasoning as they identify, compare, and classify two- or three-dimensional geometric figures; and they use numbers, standard units, and measurement tools to describe and compare objects, make estimates, and solve application problems. Students organize data, choose an appropriate method to display the data, and interpret the data to make decisions and predictions and solve problems.
  - (3) Throughout mathematics in Grades 3-5, students develop numerical fluency with conceptual understanding and computational accuracy. Students in Grades 3-5 use knowledge of the base-ten place value system to compose and decompose numbers in order to solve problems requiring precision, estimation, and reasonableness. By the end of Grade 5, students know basic addition, subtraction, multiplication, and division facts and are using them to work flexibly, efficiently, and accurately with numbers during addition, subtraction, multiplication, and division computation.
  - (4) Problem solving, language and communication, connections within and outside mathematics, and formal and informal reasoning underlie all content areas in mathematics. Throughout mathematics in Grades 3-5, students use these processes together with technology and other mathematical tools such as manipulative materials to develop conceptual understanding and solve meaningful problems as they do mathematics.
- (b) Knowledge and skills.
  - (1) Number, operation, and quantitative reasoning. The student uses place value to represent whole numbers and decimals. The student is expected to:
    - (A) use place value to read, write, compare, and order whole numbers through 999,999,999; and
    - (B) use place value to read, write, compare, and order decimals involving tenths and hundredths, including money, using concrete objects and pictorial models.
  - (2) Number, operation, and quantitative reasoning. The student describes and compares fractional parts of whole objects or sets of objects. The student is expected to:
    - (A) use concrete objects and pictorial models to generate equivalent fractions;
    - (B) model fraction quantities greater than one using concrete objects and pictorial models;
    - (C) compare and order fractions using concrete objects and pictorial models; and
    - (D) relate decimals to fractions that name tenths and hundredths using concrete objects and pictorial models.
  - (3) Number, operation, and quantitative reasoning. The student adds and subtracts to solve meaningful problems involving whole numbers and decimals. The student is expected to:
    - (A) use addition and subtraction to solve problems involving whole numbers; and
    - (B) add and subtract decimals to the hundredths place using concrete objects and pictorial models.

- (4) Number, operation, and quantitative reasoning. The student multiplies and divides to solve meaningful problems involving whole numbers. The student is expected to:
  - (A) model factors and products using arrays and area models;
  - (B) represent multiplication and division situations in picture, word, and number form;
  - (C) recall and apply multiplication facts through 12 x 12;
  - (D) use multiplication to solve problems (no more than two digits times two digits without technology); and
  - (E) use division to solve problems (no more than one-digit divisors and three-digit dividends without technology).
- (5) Number, operation, and quantitative reasoning. The student estimates to determine reasonable results. The student is expected to:
  - (A) round whole numbers to the nearest ten, hundred, or thousand to approximate reasonable results in problem situations; and
  - (B) use strategies including rounding and compatible numbers to estimate solutions to multiplication and division problems.
- (6) Patterns, relationships, and algebraic thinking. The student uses patterns in multiplication and division. The student is expected to:
  - (A) use patterns and relationships to develop strategies to remember basic multiplication and division facts (such as the patterns in related multiplication and division number sentences (fact families) such as  $9 \ge 81$  and  $81 \div 9 = 9$ ); and
  - (B) use patterns to multiply by 10 and 100.
- (7) Patterns, relationships, and algebraic thinking. The student uses organizational structures to analyze and describe patterns and relationships. The student is expected to describe the relationship between two sets of related data such as ordered pairs in a table.
- (8) Geometry and spatial reasoning. The student identifies and describes attributes of geometric figures using formal geometric language. The student is expected to:
  - (A) identify and describe right, acute, and obtuse angles;
  - (B) identify and describe parallel and intersecting (including perpendicular) lines using concrete objects and pictorial models; and
  - (C) use essential attributes to define two- and three-dimensional geometric figures.
- (9) Geometry and spatial reasoning. The student connects transformations to congruence and symmetry. The student is expected to:
  - (A) demonstrate translations, reflections, and rotations using concrete models;
  - (B) use translations, reflections, and rotations to verify that two shapes are congruent; and
  - (C) use reflections to verify that a shape has symmetry.
- (10) Geometry and spatial reasoning. The student recognizes the connection between numbers and their properties and points on a line. The student is expected to locate and name points on a number line using whole numbers, fractions such as halves and fourths, and decimals such as tenths.
- (11) Measurement. The student applies measurement concepts. The student is expected to estimate and measure to solve problems involving length (including perimeter) and area. The student uses measurement tools to measure capacity/volume and weight/mass. The student is expected to:
  - (A) estimate and use measurement tools to determine length (including perimeter), area, capacity and weight/mass using standard units SI (metric) and customary;

- (B) perform simple conversions between different units of length, between different units of capacity, and between different units of weight within the customary measurement system;
- (C) use concrete models of standard cubic units to measure volume;
- (D) estimate volume in cubic units; and
- (E) explain the difference between weight and mass.
- (12) Measurement. The student applies measurement concepts. The student measures time and temperature (in degrees Fahrenheit and Celsius). The student is expected to:
  - (A) use a thermometer to measure temperature and changes in temperature; and
  - (B) use tools such as a clock with gears or a stopwatch to solve problems involving elapsed time.
- (13) Probability and statistics. The student solves problems by collecting, organizing, displaying, and interpreting sets of data. The student is expected to:
  - (A) use concrete objects or pictures to make generalizations about determining all possible combinations of a given set of data or of objects in a problem situation; and
  - (B) interpret bar graphs.
- (14) Underlying processes and mathematical tools. The student applies Grade 4 mathematics to solve problems connected to everyday experiences and activities in and outside of school. The student is expected to:
  - (A) identify the mathematics in everyday situations;
  - (B) solve problems that incorporate understanding the problem, making a plan, carrying out the plan, and evaluating the solution for reasonableness;
  - (C) select or develop an appropriate problem-solving plan or strategy, including drawing a picture, looking for a pattern, systematic guessing and checking, acting it out, making a table, working a simpler problem, or working backwards to solve a problem; and
  - (D) use tools such as real objects, manipulatives, and technology to solve problems.
- (15) Underlying processes and mathematical tools. The student communicates about Grade 4 mathematics using informal language. The student is expected to:
  - (A) explain and record observations using objects, words, pictures, numbers, and technology; and
  - (B) relate informal language to mathematical language and symbols.
- (16) Underlying processes and mathematical tools. The student uses logical reasoning. The student is expected to:
  - (A) make generalizations from patterns or sets of examples and nonexamples; and
  - (B) justify why an answer is reasonable and explain the solution process.

Source: The provisions of this §111.16 adopted to be effective September 1, 1998, 22 TexReg 7623; amended to be effective August 1, 2006, 30 TexReg 7471.

## §111.17. Mathematics, Grade 5.

- (a) Introduction.
  - (1) Within a well-balanced mathematics curriculum, the primary focal points at Grade 5 are comparing and contrasting lengths, areas, and volumes of two- or three-dimensional geometric figures; representing and interpreting data in graphs, charts, and tables; and applying whole number operations in a variety of contexts.

- (2) Throughout mathematics in Grades 3-5, students build a foundation of basic understandings in number, operation, and quantitative reasoning; patterns, relationships, and algebraic thinking; geometry and spatial reasoning; measurement; and probability and statistics. Students use algorithms for addition, subtraction, multiplication, and division as generalizations connected to concrete experiences; and they concretely develop basic concepts of fractions and decimals. Students use appropriate language and organizational structures such as tables and charts to represent and communicate relationships, make predictions, and solve problems. Students select and use formal language to describe their reasoning as they identify, compare, and classify two- or three-dimensional geometric figures; and they use numbers, standard units, and measurement tools to describe and compare objects, make estimates, and solve application problems. Students organize data, choose an appropriate method to display the data, and interpret the data to make decisions and predictions and solve problems.
- (3) Throughout mathematics in Grades 3-5, students develop numerical fluency with conceptual understanding and computational accuracy. Students in Grades 3-5 use knowledge of the base-ten place value system to compose and decompose numbers in order to solve problems requiring precision, estimation, and reasonableness. By the end of Grade 5, students know basic addition, subtraction, multiplication, and division facts and are using them to work flexibly, efficiently, and accurately with numbers during addition, subtraction, multiplication, and division facts.
- (4) Problem solving, language and communication, connections within and outside mathematics, and formal and informal reasoning underlie all content areas in mathematics. Throughout mathematics in Grades 3-5, students use these processes together with technology and other mathematical tools such as manipulative materials to develop conceptual understanding and solve meaningful problems as they do mathematics.

## (b) Knowledge and skills.

- (1) Number, operation, and quantitative reasoning. The student uses place value to represent whole numbers and decimals. The student is expected to:
  - (A) use place value to read, write, compare, and order whole numbers through the 999,999,999,999; and
  - (B) use place value to read, write, compare, and order decimals through the thousandths place.
- (2) Number, operation, and quantitative reasoning. The student uses fractions in problem-solving situations. The student is expected to:
  - (A) generate a fraction equivalent to a given fraction such as 1/2 and 3/6 or 4/12 and 1/3;
  - (B) generate a mixed number equivalent to a given improper fraction or generate an improper fraction equivalent to a given mixed number;
  - (C) compare two fractional quantities in problem-solving situations using a variety of methods, including common denominators; and
  - (D) use models to relate decimals to fractions that name tenths, hundredths, and thousandths.
- (3) Number, operation, and quantitative reasoning. The student adds, subtracts, multiplies, and divides to solve meaningful problems. The student is expected to:
  - (A) use addition and subtraction to solve problems involving whole numbers and decimals;
  - (B) use multiplication to solve problems involving whole numbers (no more than three digits times two digits without technology);
  - (C) use division to solve problems involving whole numbers (no more than two-digit divisors and three-digit dividends without technology), including interpreting the remainder within a given context;
  - (D) identify common factors of a set of whole numbers; and

- (E) model situations using addition and/or subtraction involving fractions with like denominators using concrete objects, pictures, words, and numbers.
- (4) Number, operation, and quantitative reasoning. The student estimates to determine reasonable results. The student is expected to use strategies, including rounding and compatible numbers to estimate solutions to addition, subtraction, multiplication, and division problems.
- (5) Patterns, relationships, and algebraic thinking. The student makes generalizations based on observed patterns and relationships. The student is expected to:
  - (A) describe the relationship between sets of data in graphic organizers such as lists, tables, charts, and diagrams; and
  - (B) identify prime and composite numbers using concrete objects, pictorial models, and patterns in factor pairs.
- (6) Patterns, relationships, and algebraic thinking. The student describes relationships mathematically. The student is expected to select from and use diagrams and equations such as y = 5 + 3 to represent meaningful problem situations.
- (7) Geometry and spatial reasoning. The student generates geometric definitions using critical attributes. The student is expected to identify essential attributes including parallel, perpendicular, and congruent parts of two- and three-dimensional geometric figures.
- (8) Geometry and spatial reasoning. The student models transformations. The student is expected to:
  - (A) sketch the results of translations, rotations, and reflections on a Quadrant I coordinate grid; and
  - (B) identify the transformation that generates one figure from the other when given two congruent figures on a Quadrant I coordinate grid.
- (9) Geometry and spatial reasoning. The student recognizes the connection between ordered pairs of numbers and locations of points on a plane. The student is expected to locate and name points on a coordinate grid using ordered pairs of whole numbers.
- (10) Measurement. The student applies measurement concepts involving length (including perimeter), area, capacity/volume, and weight/mass to solve problems. The student is expected to:
  - (A) perform simple conversions within the same measurement system (SI (metric) or customary);
  - (B) connect models for perimeter, area, and volume with their respective formulas; and
  - (C) select and use appropriate units and formulas to measure length, perimeter, area, and volume.
- (11) Measurement. The student applies measurement concepts. The student measures time and temperature (in degrees Fahrenheit and Celsius). The student is expected to:
  - (A) solve problems involving changes in temperature; and
  - (B) solve problems involving elapsed time.
- (12) Probability and statistics. The student describes and predicts the results of a probability experiment. The student is expected to:
  - (A) use fractions to describe the results of an experiment;
  - (B) use experimental results to make predictions; and
  - (C) list all possible outcomes of a probability experiment such as tossing a coin.
- (13) Probability and statistics. The student solves problems by collecting, organizing, displaying, and interpreting sets of data. The student is expected to:
  - (A) use tables of related number pairs to make line graphs;

- (B) describe characteristics of data presented in tables and graphs including median, mode, and range; and
- (C) graph a given set of data using an appropriate graphical representation such as a picture or line graph.
- (14) Underlying processes and mathematical tools. The student applies Grade 5 mathematics to solve problems connected to everyday experiences and activities in and outside of school. The student is expected to:
  - (A) identify the mathematics in everyday situations;
  - (B) solve problems that incorporate understanding the problem, making a plan, carrying out the plan, and evaluating the solution for reasonableness;
  - (C) select or develop an appropriate problem-solving plan or strategy, including drawing a picture, looking for a pattern, systematic guessing and checking, acting it out, making a table, working a simpler problem, or working backwards to solve a problem; and
  - (D) use tools such as real objects, manipulatives, and technology to solve problems.
- (15) Underlying processes and mathematical tools. The student communicates about Grade 5 mathematics using informal language. The student is expected to:
  - (A) explain and record observations using objects, words, pictures, numbers, and technology; and
  - (B) relate informal language to mathematical language and symbols.
- (16) Underlying processes and mathematical tools. The student uses logical reasoning. The student is expected to:
  - (A) make generalizations from patterns or sets of examples and nonexamples; and
  - (B) justify why an answer is reasonable and explain the solution process.

Source: The provisions of this §111.17 adopted to be effective September 1, 1998, 22 TexReg 7623; amended to be effective August 1, 2006, 30 TexReg 7471.